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OVERLAP FUNCTIONS IN NUCLEAR CORRELATION METHODS AND DIRECT NUCLEON REMOVAL PROCESSES

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A theoretical method to obtain overlap functions and spectroscopic factors from a model one-body density matrix (OBDM) accounting for short-range nucleon-nucleon correlations is applied to describe one-nucleon removal processes on the ^{16}O and ^{40}Ca nuclei. The method allows the differential cross sections of (*p*, *d*) reactions and the momentum distributions of a transitions to a single-particle states from (*e*, *e′p*) reactions to be calculated. It is shown that the overlap functions obtained within the Jastrow correlation method lead to a satisfactory description of the quantities considered.

Функции перекрытия в ядерных корреляционных методах и прямые процессы с перемещением одного нуклона

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Теоретический метод получения функций перекрытия и спектроскопических факторов из модельной одночастичной матрицы плотности, учитывающий короткодействующие нуклон-нуклонные корреляции, используется для описания процессов с выбиванием одного нуклона из ядер ^{16}O и ^{40}Ca. Вычислены соответствующие дифференциальные сечения (*p*, *d*)-реакций, а также импульсные распределения переходов к одночастичным состояниям из (*e*, *e′p*)-реакций. Сравнение с экспериментом показало, что полученные функции перекрытия с учетом короткодействующих корреляций Ястрова позволяют удовлетворительно описать рассматриваемые величины.

1. Introduction

The one-nucleon removal reactions provide useful information about the nuclear structure and the mechanisms of the interaction processes. Such reactions have been used to extract spectroscopic factors from the observed reaction cross sections. The (*p*, *d*) reaction has been a reliable spectroscopic tool (see, e.g., [1–7]). Knock-out reactions initiated with electrons have been also used for studying single-particle properties of nuclei (e.g., [8–12]).

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In particular, the extracted data for the nucleon momentum distributions from the (*e*, *e'p*) experiments for a variety of nuclei by means of the *y*-scaling method [13] show unambiguously the existence of high-momentum components in them.

In the one-nucleon removal reactions the measured momentum distribution for a transition to a discrete state α in the residual nucleus $\rho_\alpha(p_m)$ is expressed by the bound-state wave function which is the Fourier transform of the overlap wave function $\phi_\alpha(\mathbf{r})$ between the ground state wave function of the target $\Psi^{(A)}$ and the wave function of the final state of the residual nuclues $\Psi^{(A-1)}$ [14,15]. In the coordinate space $\phi_\alpha(\mathbf{r})$ is defined as

$$\phi_\alpha(\mathbf{r}) = \langle \Psi_\alpha^{(A-1)} | a(\mathbf{r}) | \Psi^{(A)} \rangle, \tag{1}$$

where $a^\dagger(\mathbf{r})$ and $a(\mathbf{r})$ are creation and annihilation operators for a nucleon with spatial coordinate **r** (spin and isospin coordinates are not put in evidence). In the mean-filed approximation (MFA) $\Psi^{(A)}$ and $\Psi^{(A-1)}$ are Slater determinants and the overlap wave function is identified with the single-particle wave function corresponding to the mean-field potential. In approaches going beyond the limits of the MFA (correlation methods) the overlap function is different from the MFA single-particle wave function. Therefore, the growing interest in the interpretation of the recent (*p*, *d*) and (*e*, *e'p*) experimental data is motivated by the possibility to clarify the limitation of the nuclear mean-field picture and to find the extent to which the hole orbitals in nuclei are depleted by means of the experimentally based information. The overlap functions (1) are not orthonormalized. Their norm defines the spectroscopic factor of the level α

$$S_\alpha = \langle \phi_\alpha | \phi_\alpha \rangle \tag{2}$$

and the normalized overlap function is:

$$\tilde{\phi}_\alpha(\mathbf{r}) = S_\alpha^{-1/2} \phi_\alpha(\mathbf{r}). \tag{3}$$

The general relationship which connects the asymptotic behaviour of the one-body density matrix with the overlap functions of the (*A* – 1)-particle system eigenstates [16] is of significant importance because it enables one to obtain quantities connected with them by means of the exact OBDM (or by a realistic one obtained with a given correlation method) of the ground state of the *A*-particle system. The one-body density matrix associated with the ground state $\Psi^{(A)}$ of the target nucleus with *A* nucleons is defined as

$$\rho(\mathbf{r}, \mathbf{r}') = \langle \Psi^{(A)} | a^\dagger(\mathbf{r})\, a(\mathbf{r}') | \Psi^{(A)} \rangle. \tag{4}$$

After inserting the complete set of eigenstates $\Psi_\alpha^{(A-1)}$ of the residual (*A* – 1)-nucleus, Eq.(4) becomes:

$$\rho(\mathbf{r}, \mathbf{r}') = \sum_\alpha \phi_\alpha^*(\mathbf{r})\, \phi_\alpha(\mathbf{r}') = \sum_\alpha S_\alpha \tilde{\phi}_\alpha^*(\mathbf{r})\, \tilde{\phi}_\alpha(\mathbf{r}'), \tag{5}$$

where ϕ_α and $\tilde{\phi}_\alpha$ are the overlap functions (1) and (3), respectively, S_α is the spectroscopic factor (2) and the summation implicitly includes also the continuum states associated with all scattering channels of the $(A-1)$ system.

The aim of the present work is to test the applicability of the overlap functions, obtained from the asymptotic restoration procedure [17] (which uses a correlated OBDM from the Jastrow correlation method [18,19]) as realistic form factors within the DWBA analyses of pick-up reactions on ^{16}O and ^{40}Ca nuclei. We emphasize that our calculations give *absolute cross sections* with no normalizing factors. In addition, the theoretical results for the single-particle momentum distributions are compared with the available experimental ones from $(e, e'p)$ reactions on the same nuclei.

The results for the differential cross sections of (p, d)-pick-up reactions calculated within the DWBA with overlap functions as trial form factors are presented in Sec.2. In Sec.3 a comparison of the theoretical estimations for the single-particle nucleon momentum distributions with available experimental data for $(e, e'p)$-knock-out reactions is made. The concluding remarks are given in Sec.4.

2. Overlap Functions as Realistic Form Factors in the (p, d)-Pick-Up Reactions

It has been shown in [19] that the nucleon-nucleon correlations affect mainly the one-body characteristics of the particle states above the Fermi level, while the hole states are almost unaffected. Moreover, this fact was confirmed in [17] using the possibility to restore the overlap functions, the separation energies and the spectroscopic factors for bound $(A-1)$-particle eigenstates by means of the ground-state one-body density matrix of the target A-particle system. The overlap-, mean-field- and natural orbital wave functions are quite similar for the hole states in nuclei. This justifies the use of shell-model orbitals instead of overlap functions within DWBA calculations for such kind of nuclear states. This approximation, however, is no longer valid for the particle states where the overlap functions significantly differ from the mean-field wave functions. Therefore, neither natural orbitals nor shell-model wave function can be used instead of the particle-state overlap functions within the DWBA analysis of the experimental data for one-nucleon transfer reactions.

It has been shown in [16] that the radial part of the overlap function for the lowest bound state can be obtained by the radial part of the OBDM $\rho_{lj}(r, r')$ at $r' \equiv a \to \infty$:

$$\phi_{n_0 lj}(r) = \frac{\rho_{lj}(r, a)}{C_{n_0 lj} \exp(-k_{n_0 lj}\, a)/a}, \tag{6}$$

where

$$k_{nlj} = \hbar^{-1} \sqrt{2m(E_{nlj}^{(A-1)} - E_0^{(A)})} \tag{7}$$

depends on the separation energy $E_\alpha = E_{nlj}^{(A-1)} - E_0^{(A)}$. The separation energy is:

$$\varepsilon_{n_0lj} = \hbar^2 k^2_{n_0lj} / 2m \tag{8}$$

and the spectroscopic factor is:

$$S_{n_0lj} = \langle \phi_{n_0lj} | \phi_{n_0lj} \rangle. \tag{9}$$

The coefficient C_{n_0lj} can be obtained from the asymptotic form of the diagonal part of the radial OBDM. In principle, the overlap functions for all bound states of the $(A-1)$ nucleon system can be constructed from the OBDM repeating the above procedure.

In order to test the role of the overlap functions as realistic form factors we have calculated using the DWUCK4 code [20] the (p, d) reaction on ^{16}O and ^{40}Ca at different incident proton energy. The overlap functions for the one-neutron removal from $1p$ and $1d$ states in ^{16}O obtained in [17] are compared in Fig.1 with the form factors calculated under two different approximations, namely the separation energy prescription (SEP) and the effective binding energy prescription (EBEP).

The differential cross section for the (p, d)-pick-up transitions is calculated within the DWBA approach using a zero-range interaction. It can be written in the form [20]:

$$\frac{d\sigma^{lsj}_{pd}(\theta)}{d\Omega} = \frac{3}{2}\frac{S_{lsj}}{2j+1}\frac{D_0^2}{10^4}\sigma^{lsj}_{DW}(\theta), \tag{10}$$

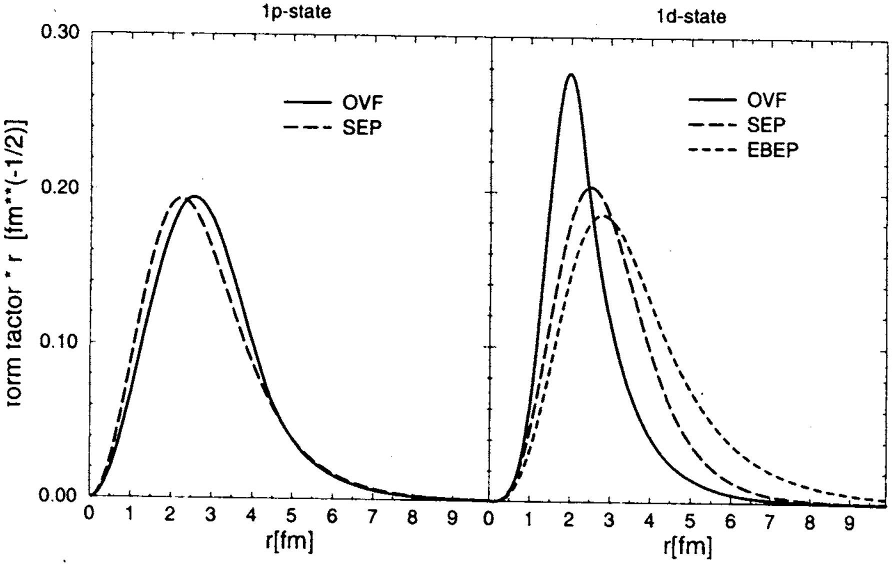

Fig.1. One-neutron removal overlap functions [17] (solid line) and form factors obtained within the SEP (long-dashed line) and EBEP (short-dashed line) for the transitions to the $1/2^-$ ground state and to the $5/2^+$ excited state in ^{15}O. All functions are normalized to unity

where S_{lsj} is the spectroscopic amplitude, j is the total angular momentum of the final state, $D_0^2 \approx 1.5 \times 10^4$ MeV·fm^3 and $\sigma_{DW}^{lsj}(\theta)$ is the cross section calculated by the DWUCK4.

The standard DWUCK4 procedure is performed by calculating the bound-neutron wave function using the SEP and EBEP and different sets of proton and deuteron optical model parameters are used. For our purposes the standard DWBA form factor was exchanged by that obtained in the framework of the one-body density matrix calculations and the spectroscopic factor S_{lsj} in (10) was taken to be equal to unity, since our overlap functions «contain» the sepctroscopic factor. Their normalization is (2):

$$4\pi \int |\phi_{lsj}(r)|^2 r^2 dr = S_{lsj}. \qquad (11)$$

As an example, the results for the differential cross sections of ^{16}O (p, d) reaction at $E_p = 45.34$ MeV calculated with overlap functions as trial form factors and with standard one are given in Fig.2. They are compared with the experimental data from [2,21]. As can be seen the shape of the angular distribution for the raction is well reproduced in both types of calculations. The use of overlap functions for the hole state leads to a good agreement with the data in their absolute values. This means that the residual interactions are taken into account sufficiently well as compared with the other models. Moreover, the spectroscopic factors deduced from the restoration procedure which are «contained» in the overlap functions are sufficient to reproduce the amplitude of the first maximum of the differential cross section for the hole states. The results for the transitions to the excited states are less

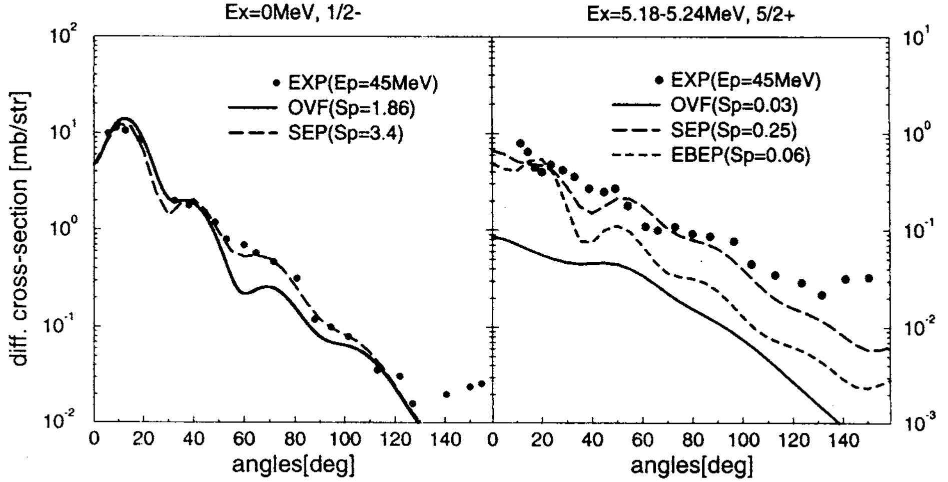

Fig.2. Differential cross section (solid line) for the ^{16}O (p, d) reaction at $E_p = 45.34$ MeV incident energy to the $1/2^-$ and $5/2^+$ states in ^{15}O. The DWBA results within the SEP (long-dashed line) and EBEP (short-dashed line) are shown. The experimental data [2,21] are given by the solid circles

Table. Spectroscopic factors for the ground states and the $5/2^+$ and $7/2^-$ excited states in ^{15}O and ^{39}Ca, respectively

^{15}O			^{39}Ca		
	$1/2^-$ (*g.s.*)	$5/2^+$		$3/2^+$ (*g.s.*)	$7/2^-$
OVF [17]	1.86	0.03	OVF [17]	3.60	0.08
SEP ($E_p = 45$ MeV)	3.40	0.25	SEP ($E_p = 27.5$ MeV)	2.30	0.32
EBEP ($E_p = 45$ MeV)		0.06	EBEP ($E_p = 27.5$ MeV)		0.16
			SEP ($E_p = 65$ MeV)	3.70	0.18
			EBEP ($E_p = 65$ MeV)		0.08

satisfactory. The reason is that the population of the particle states in ^{16}O and ^{40}Ca cannot be realistically described by the simple central pair-correlation included in the Jastrow-type OBDM.

In the Table we give the values of the spectroscopic factors of the transitions to the ground state and some excited states in ^{16}O and ^{40}Ca considered in our work. A comparison is made between the spectroscopic factors extracted from the OBDM [17] and those from the experimental data applying DWBA. Let us consider the spectroscopic factors of the transitions to the ground state, especially in ^{16}O. The problem of obtaining reasonable values for the spectroscopic factors in ^{16}O is well known and intensively studied [21,22]. The DWBA calculations with optical model parameters obtained from the elastic scattering analysis fail to reproduce the shape of the differential cross section [22]. By adopting the adiabatical deuteron optical model the shape of the cross section is well reproduced but the value of the spectroscopic factor exceeds the maximum allowed value of 2. However, using the overlap function from [17] which has a physically acceptable magnitude, one can achieve good agreement with the experimental cross section. We note that the results of the standard DWBA calculations depend significantly on the optical potentials used and on the incident energy. Let us look at the spectroscopic factors of the transitions to the excited states. It is seen that they vary among themselves more than the values for the ground states. They depend on the optical model parameters and the incident energies but also on the procedure applied to calculate the form factors. The spectroscopic factors of the overlap functions [17] do not always reproduce the magnitude of the experimental cross section. Obviously it is difficult to obtain reliable information about the absolute spectroscopic factors of the excited states unless a more refined OBDM is used.

We would like to emphasize that our calculations are absolute in contrast to the standard DWBA results which are adjusted to give the correct magnitude of the differential cross section. Especially for the particle states different prescriptions for calculating the form factor, which have acceptable justification, lead to quite different values for the spectroscopic factors. Thus it is worthwhile to investigate the differential cross sections of pick-up reactions using absolute form factors which are extracted, however, from more sophisticated one-body density matrices, as for example the ones obtained by variational Monte Carlo calculations [23].

3. Momentum Distribution of a Transition to a Single-Particle State from (*e*, *e′p*) Reactions

The most direct way to get information on the single-particle wave functions is to study the one-nucleon knock-out reactions. In this Section we present some results of the calculations of the single-particle momentum distributions corresponding to a transition to a given s.p. state in comparison with the empirical data from the (*e*, *e′p*) reactions. In the Plane-Wave Impulse Approximation (PWIA) the energy ω and the momentum $\mathbf{q}$ lost by the electron are transferred to a proton with binding (missing) energy E_m and missing momentum $\mathbf{p}_m$. From the energy and momentum conservation laws the latter are determined by

$$E_m = \omega - E_p - T_{A-1}, \qquad \mathbf{p}_m = \mathbf{k}_p - \mathbf{q}, \tag{12}$$

where $\mathbf{k}_p$ and E_p are the momentum and the energy of the knocked-out proton, respectively, and T_{A-1} is the kinetic energy of the residual nucleus. The (*e*, *e′p*) cross section in the PWIA can be written in the form:

$$\frac{d^6\sigma}{de' d\mathbf{k}_p} = K\sigma_{ep} S(\mathbf{p}_m, E_m), \tag{13}$$

where K is a kinematical factor and σ_{ep} is the elementary electron-proton cross section [24]. The spectral function $S(\mathbf{p}_m, E_m)$ is the joint probability of finding a proton with separation energy E_m and momentum $\mathbf{p}_m$ inside the nucleus. For the transition to a discrete state α one can write

$$S(\mathbf{p}_m, E_m) = \rho_\alpha(\mathbf{p}_m)\, \delta(E_m - E_\alpha), \tag{14}$$

where the s.p. momentum distribution

$$\rho_\alpha(\mathbf{p}_m) = |\phi_\alpha(\mathbf{p}_m)|^2 \tag{15}$$

is the Fourier fransform squared of the overlap (1) between the initial and final nuclear state. The integration of the empirical data over the interval that covers the peak of the transition under study gives the single-particle momentum distribution $\rho_\alpha(\mathbf{p}_m)$. The spectroscopic factor S_α for a given α-state is determined by scaling the theoretical predictions for $\rho_\alpha(\mathbf{p}_m)$ to the experimental data.

In the present work we consider the s.p. momentum distributions in the ^{16}O and ^{40}Ca nuclei. As an example, we given in Fig.3 the momentum distribution for the transition to the second $1/2^+$ excited state in ^{39}K from the ^{40}Ca(*e*, *e′p*) reaction. In Fig.3 the

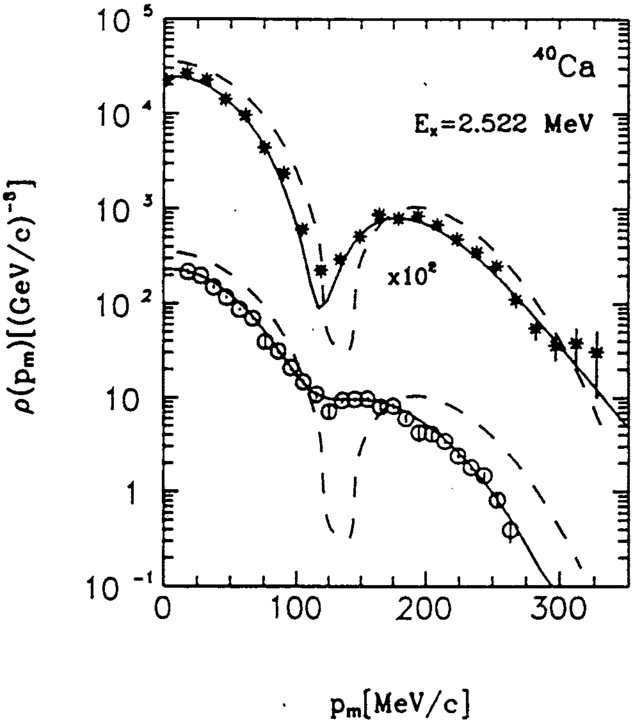

Fig.3. Momentum distribution for the transition to the second $1/2^+$ excited state in ^{39}K from ^{40}Ca $(e, e'p)$ reaction. The calculations by using 2*s*-overlap functions from [17] are presented by the dashed line. The stars and circles represent the empirical data measured in different kinematics [9]. The CDWIA calculations [9] are shown by solid lines

experimental data are given together with their Coulomb Distorted Wave Impulse Approximation (CDWIA) analysis [9] which is used to obtain spectroscopic factors for discrete transitions and rms radii for the various orbitals. The CDWIA calculations are performed employing different wave functions so that their rms radii are fitted to dscribe the data. In our method the necessity to use such parameters is avoided and we would like to emphasize the possibility of obtaining the momentum distribuitons in a consistent way on the basis of the OBDM corresponding to a correlated system. At the same time it is important to note the fact that the overlap functions from [17] are for neutron bound states. In the case of proton bound states some modifications due to the Coulomb asymptotic behaviour of the overlap functions have to be taken into account. Nevertheless, one can see that the calculated results for the momentum distributions are close to the experimental data. Especially this can be seen from Fig.3 where the minimum in the momentum distribution for the transition leading to the $1/2^+$ state in the ^{39}K at 2.522 MeV is well described. Our calculations show that generally the shape of the momentum distributions of the transitions can be adequately described by mean-field wave functions for momenta up to the Fermi momentum k_F ($\approx$ 250 MeV/c). Measurements over a more extended range of the momenta and with better accuracy are necessary to test the various bound-state wave functions.

4. Conclusions

In the present work a new method for theoretical studies of the one-nucleon removal processes is suggested. It is shown that the overlap functions of the $(A-1)$-particle system eigenstates obtained in [17] by using the general relationship [16] which connects the asymptotic behaviour of the OBDM with them can be successfully applied as realistic form factors to calculate the differential cross sections of ^{16}O (*p*, *d*)- and ^{40}Ca (*p*, *d*)-pick-up reactions at various incident energies. The results of our calculations demonstrate that the obtained angular distributions do not need remormalization by introducing the spectroscopic factors. The overlap functions provide a physical and model-independent definition of the spectroscopic factors. The angular distributions are in good agreement with the experimental data in their absolute values for the ground states and less satisfactory for the excited states.

Our calculations of the s.p. momentum distributions of various orbitals in ^{16}O and ^{40}Ca describe satisfactorily the empirical data obtained by the (*e*, *e'p*)-knock-out reactions at momenta below the Fermi momentum. The results are also in agreement with the CDWIA calculations using bound-state wave functions evaluated in a mean-field potential. New measurements should be done at higher momenta where the SRC effects on the momentum distribution are sizable. Using the correct asymptotic behaviour of the proton overlap function in the procedure [17] and more realistic OBDM, one can expect that the resulting overlap functions will be able to describe more accurately the experimental single-particle momentum distributions.

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